A.1.1 Mixed logistic regression; a WinBUGS comparison

Mixed regression regression models will usually have a block diagonal Hessian du to grouping/clustering of the data. The present model was deliberately chosen not to be separable, in order to pose a computational challenge to both ADMB-RE and WinBUGS.

Model description Let $\mathbf{y} = (y_1, \ldots, y_n)$ be a vector of dichotomous observations ($y_i \in \{0, 1\}$), and let $\mathbf{u} = (u_1, \ldots, u_q)$ be a vector of independent random effects, each with Gaussian distribution (expectation 0 and variance σ^2). Define the success probability $\pi_i = \Pr(y_i = 1)$. The following relationship between π_i and explanatory variables (contained in matrices $\mathbf{X}$ and $\mathbf{Z}$) is assumed:

$$\log\left(\frac{\pi_i}{1-\pi_i}\right) = \mathbf{X}_i\beta + \mathbf{Z}_i\mathbf{u},$$

where $\mathbf{X}_i$ and $\mathbf{Z}_i$ are the i'th rows of the known covariates matrices $\mathbf{X}$ $(n \times p)$ and $\mathbf{Z}$ $(n \times q)$, respectively, and β is a p-vector of regression parameters. Thus, the vector of fixed-effects vector is $\theta = (\beta, \log\sigma)$.

Results The goal here is to compare computation times with BUGS on a simulated data set. For this purpose we use $n = 200$, $p = 5$, $q = 30$, and values of the the hyper parameters as shown in the table below ('True values'). The matrices $\mathbf{X}$ and $\mathbf{Z}$ were generated randomly with each element uniformly distributed on $[-2, 2]$. As start values for both AD Model Builder and BUGS we used $\beta_{\text{init},j} = -1$ and $\sigma_{\text{init}} = 4.5$. In BUGS we used a uniform $[-10, 10]$ prior on β_j and a standard (in the BUGS literature) noninformative gamma prior on $\tau = \sigma^{-2}$. In AD Model Builder the parameter bounds $\beta_j \in [-10, 10]$ and $\log\sigma \in [-5, 3]$ were used in the optimization process.

	β_1	β_2	β_3	β_4	β_5	σ
True values	0.0000	0.0000	0.0000	0.0000	0.0000	0.1000
ADMB-RE	0.0300	-0.0700	0.0800	0.0800	-0.1100	0.1700
Std. dev.	0.1500	0.1500	0.1500	0.1400	0.1600	0.0500
WinBUGS	0.0390	-0.0787	0.0773	0.0840	-0.1041	0.1862

On the simulated dataset AD Model Builder used 27 seconds to converge to the optimum of likelihood surface. On the same dataset we first ran WinBUGS (Version 1.4) for 5,000 iterations. The recommended convergence diagnostic in WinBUGS is the Gelman-Rubin plot (see the help files available from the

menues in WinBUGS) which require that two Markov chains are run in parallel. From the Gelman-Rubin plot it was clear that convergence appeared after approximately $2,000$ iterations. The time taken by WinBUGS to perform generate the first $2,000$ was approximately 700 seconds.

Files http://otter-rsch.com/admbre/examples/logistic/logistic.html

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